The physics and mathematics of javelin throwing

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Abstract

Javelin flight is strictly governed by the laws of aerodynamics but there remain well-entrenched but misleading views about the factors involved. This paper gives a simplified but accurate view of the physics and dynamics of javelin flight and describes a software implementation of these.

1 Overview

This paper was written after realising that there were still many misunderstandings about the actual physics of throwing in general and javelin throwing in particular. I am nowadays a computer scientist and UKA level 2 javelin coach but was a fairly competent thrower in my youth, so this may help both coaches and athletes to understand what is going on. Even if you don't understand the maths, the accompanying software which will run on a PC will still help you simply by playing with it and watching what happens.

The software will only model post-1986 javelins. The essential difference was that the centre of gravity was moved by 10cm in 1986 because of the prodigious distances being achieved by such throwers as Uwe Hohn, ($>$ 104m.). The only other alternative would have been to move the javelin outside the main arena which would have deprived onlookers of seeing one of the most spectacular field events. The effect of this re-balancing is essentially two-fold:-

- The distance is reduced by around 10%
- Moving the centre of gravity forward means that it is now about 6cm in front of the *centre of pressure*. The centre of pressure is defined to be the point at which the aerodynamic forces of lift and drag on the javelin apply. This means that there is an upward lifting force 6cm aft of the downward force acting through the centre of gravity which is situated around the front edge of the handle. The effect is that the javelin experiences a turning moment in the vertical plane which forces the point down and which therefore causes it to stick in removing any ambiguity as to where it has first landed. The women's 600gm javelin was similarly adjusted about 5 years ago for similar reasons. Unfortunately, the disastrous current

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performance of many 700 gm javelins is because no such steps have been taken with this javelin with the result that the manufacturers seem to ignore it presumably to promote greater distances. Unfortunately, the flight characteristics would suggest that the centre of gravity and centre of pressure are effectively co-located meaning that it will very frequently land flat and I have personally witnessed flat landings at 60m in the English Schools with this most frustrating implement as well as half of all throws in the British Master's championships being flat landers, (M50 and M55 use the 700gm.

2 Some basic principles

The javelin is not so much a throw as a long pull. This is a most important lesson to impart to young athletes as they gradually begin to pick the event up. The *only* thing that matters is the speed of the javelin as it leaves the hand, assuming it is aligned properly. Furthermore, the distance it travels is proportional to the square of the speed so that an increase of 10% in the speed increases the distance by a factor of 1.1 x 1.1 = 1.21 or 21%. Of course increasing speed by 10% is harder than it sounds and is dependent on the biomechanical properties of the thrower.

So how is the javelin accelerated up to its final speed before leaving the hand ? In essence, the hand pulls on the javelin for a distance of some 2m in adult champion throwers accelerating the javelin from around 6 metres per second relative to the ground (allowing for the run-up) to around 30 metres per second relative to the ground. Note that Newton's law determines how successful this is as it is not simply the force which is applied. The musculature has to accelerate not only the javelin but also the athlete's arm and upper body. Newton's law states that the acceleration achieved is the force the athlete can apply divided by the mass which is being accelerated. If the athlete has a heavy musculature, the force which can be applied has to overcome the inertia of this mass also. This is why good javelin throwers are usually long and rangy. With respect to the muscles used in the upper arm, the bicep is simply added mass which hardly contributes as the bulk of the acceleration in this phase of the throw is applied by the tricep, so big biceps equal wasted mass.

3 The dynamics of the throw

Figure 1 shows the nomenclature used.

3.1 Simple mathematical treatment

Let t_u be the time taken to reach the highest point of the flight from leaving the hand. Let t_d be the time taken to reach the ground from the highest point of the flight. Let s_u be the height reached above the point of release and let g be the acceleration due to gravity. Let h be the height at which the javelin is released. Let r be the distance covered by the javelin.

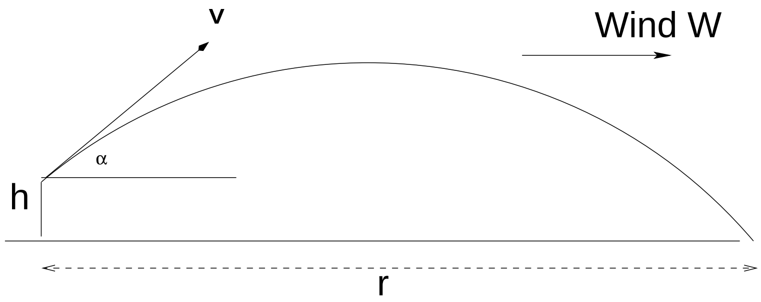

Figure 1: The nomenclature used in the accompanying text. A javelin is launched with velocity V at angle α from a height h m. above the ground with a wind of W m/s and flies a distance of r m.

Then, resolving upward, the time taken to reach maximum height resisted by gravity is given by

$$0 = V sin\alpha - gt_u \tag{1}$$

Knowing V, α and g then gives t_u. The maximum height reached above the hand is given by:-

$$0 = V^2 sin^2\alpha - 2gs_u \tag{2}$$

The time taken to reach the ground from maximum height is given by

$$s_u + h = 0 + \frac{1}{2}gt_d^2 \tag{3}$$

Knowing h, s_u and g then gives t_d. The total time the javelin is in the air is therefore $t_u + t_d$. The horizontal distance covered by the javelin is therefore given by:-

$$r = V sin\alpha(t_d + t_u) \tag{4}$$

This gives the classic parabolic shape of an ideal body in flight and is a useful approximation to the flight of a javelin. To do it more accurately however and account properly for wind and all the other factors, the aerodynamics must be re-introduced.

3.2 Enhanced mathematical treatment including aerodynamics

Linearised theory for airflow around an object like a javelin ([3]) allows us to approximate the drag force when the javelin is parallel to the wind as

$$D = 2\gamma\pi\rho V^2\epsilon^2 \tag{5}$$

where ϵ is the radius of the head of the javelin. Yaw (or attack) can be simulated by increasing this value to represent more of the javelin's aspect being presented to the air-flow around it. Various factors now come into play. Javelins come in basically three types, Headwind, Tailwind and Neither (General). Javelin manufacturers are not allowed to move the centre of gravity and it is checked at weigh-in. The only remaining parameter they can play with is then the centre of pressure. In the mid-80s one attempt to influence this was made by producing roughened tails which moves the centre of pressure in such a way as to simulate javelins similar to the pre-1986 types. Since this undermined the whole purpose of the 1986 re-modelling, it was quickly banned. Nowadays only modest changes can be made which are related to the nature of the prevailing wind. To summarise, the three kinds of javelin have the following properties:-

- Headwind. Pointed javelins which will, other things being equal fly a little further into a headwind but will not fly as well as a tailwind javelin into a tailwind.
- Tailwind. Blunter nosed javelins to attempt to break the boundary layer flow around the javelin a little earlier with the effect of bringing the centre of pressure a little closer to the centre of mass giving a smaller pitching moment to keep the nose up a little longer.
- General. These javelins are usually pointed and are not optimised in any particular way.

Folklore about the behaviour of these javelins abounds but I am not aware of any careful calibration experiments to resolve this. The model which follows shows natural small differences so clearly the aerodynamics does something.

Let m be the mass of the javelin. δ be the attack angle above the angle of delivery and let the position vector of the javelin relative to the delivery line be (r,s,q) where r is in the direction of the javelin flight, s is upwards and q is across from right to left. Resolving in each of these three directions using Newton's laws and including the drag term leads to the following differential equations:-

$$m\frac{d^2s}{dt^2} = -mg - \gamma(1 - \beta sin(\delta)((\frac{dr}{dt})^2 + (\frac{ds}{dt})^2) \tag{6}$$

$$m\frac{d^2r}{dt^2} = -\gamma(1 + \beta sin(\delta)((\frac{dr}{dt})^2 + (\frac{ds}{dt})^2) \tag{7}$$

$$m\frac{d^2q}{dt^2} = 0 \tag{8}$$

Here β and γ are fitting constants. These are integrated forward in time using a simple Kutta-Merson procedure, see for example, [2] to give the position vector (r,s,q) at all times after launch. The initial conditions at t = 0 are:-

$$r = 0 \tag{9}$$

$$\frac{dr}{dt} = V cos(\alpha) \tag{10}$$

$$s = h \tag{11}$$

$$\frac{ds}{dt} = V sin(\alpha) \tag{12}$$

$$q = 0 \tag{13}$$

$$\frac{dq}{dt} = W_x \tag{14}$$

where W_x is the crosswind.

Pitching moment Finally the pitching moment is included by basically solving:-

$$r\frac{d\alpha}{dt} = -moment \tag{15}$$

concurrently with the equations above, where the moment is the moment of the lifting and drag forces about the centre of gravity.

Rotation One factor I have left out because it appears to be small is the axial rotation of the javelin. For a right hander, coupled with the pitching moment, the javelin will be deflected slightly to the right. It is relatively easy to incorporate this and I will do at some stage.

Stiffness One other factor I have not incorporated is the effect of stiffness of the javelin. If you mishit a bendy javelin, a significant amount of the power is expended in oscillating the javelin. For a stiff javelin, this will be a lesser effect. If you don't mishit it, it doesn't matter. I will probably study the effects of this in a later version.

4 Conclusion

The above set of equations handle the qualitative behaviour of a javelin well and are included in the software package *Javelin Flight Analyser*, [1].

References

[1] Hatton, Les (2005) *Javelin Flight Analyser*, http://www.leshatton.org/javelin_2005.html

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[3] van Dyke, Milton (1970) *Perturbation methods in fluid mechanics*, Academic Press, ISBN 0-07-707640-0.