A mechanism for the invasion of a sub-tropical high by a hurricane: Part 1, static analysis

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Abstract

The major point of this paper is to show the existence of a mechanism whereby hurricanes can be attracted by an anticyclonic high and so can traverse right across them. It builds on the work of [5] and [3]. This paper considers the forces on a circular vortex in the field of a line vortex and shows the existence of an appropriate mechanism. A second paper deals with the motion of a circular vortex near a line vortex and argues that this motion is analogous to that of a hurricane near a sub-tropical high.

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1 Overview

Most tropical storms have small lateral extent compared with the weather systems they move amongst. In addition, low-latitude disturbances tend to persist in a quasi-steady state for extended periods of time and frequently greater than a week. This has been at the heart of successes in applying steering techniques over the years to help predict the track of such storms. An early example of this is shown in [5], where the motion of a tropical storm in a superimposed current is analysed and a sinusoidal motion predicted. This sinusoidal movement of a storm in a superimposed southerly current has been observed in real hurricane tracks in the Atlantic as they move towards the West Indies.

One property of tropical storm movement which we will concentrate on here is that rather than orbiting around an anticyclonic high, they occasionally traverse straight across. We will show here that there is in fact a force of attraction mechanism which could cause this. In a second paper, we will go on to analyse the motion of a hurricane in the presence of a sub-tropical high.

2 The force exerted on a circular vortex by a line vortex

We will use the circle theorem to construct the complex potential of a line vortex with circulation K (positive in the anticlockwise direction as in a sub-tropical

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high) outside the two-dimensional projection of a cylinder of radius R, initially with no circulation defined and then later with circulation. The circle theorem states that we can introduce a circular cylinder of radius R into a flow whose complex potential is f(z) provided that all the singularities associated with f(z) are outside the circle $z = |R|$ and that we add in a flow whose complex potential is $f(\frac{R^2}{z})$. This satisfies the necessary criteria:-

- The perturbation potential has all its singularities inside $z = |R|$
- The perturbation potential must $\longrightarrow 0$ as $|z| \longrightarrow \infty$
- The outside of the circle must be part of a streamline

The rationale here is that a hurricane is such an intense vortex that it behaves effectively like the cylinder in this flow regime in that there is assumed to be relatively little mixing across the intense vortex core. Models of two cell vortices suggest that the radial velocity is zero in between the two cells, [2].

To calculate the forces acting on the cylinder, we use the Blasius theorem which is

$$X - iY = \frac{1}{2} i\rho \int_C (\frac{dW}{dz})^2 dz \tag{1}$$

where X, Y are the forces per unit length in the x and y directions, ρ is the density of the fluid and W is the complex potential, [1].

2.1 No circulation on the circle

The complex potential of a line vortex at z=r is iK ln(z-r) and so for the situation shown in Figure 1 assuming no circulation on the cylinder, the complex potential is given by

$$W = \frac{iK}{2\pi}\{ln(z-r) - ln(\frac{R^2}{z} - r)\} \tag{2}$$

Regrouping and differentiating, we get

$$\frac{dW}{dz} = \frac{iK}{2\pi}\{\frac{1}{z-r} + \frac{1}{z} + \frac{1}{\frac{R^2}{r} - z}\} \tag{3}$$

Let $\overline{R} = \frac{R^2}{r}$. Then

$$(\frac{dW}{dz})^2 = -\frac{K^2}{4\pi^2}\{(\frac{1}{z-r})^2 + (\frac{1}{z})^2 + (\frac{1}{\overline{R}-z})^2 + \frac{2}{z(z-r)} + \frac{2}{(z-r)(\overline{R}-z)} + \frac{2}{z(\overline{R}-z)}\} \tag{4}$$

We want the forces on the cylinder, so we choose a contour C of radius R surrounding z=0 as shown in Figure 1 noting that z=r lies outside this and $\overline{R}$ lies inside it. Using the residue theorem, the poles of order 2 disappear and the contribution from the singularities at z=r are excluded and we are left with

$$(\frac{dW}{dz})^2 = 2\pi i\{-\frac{2}{r} - \frac{2}{\overline{R}-r}\}\frac{K^2}{4\pi^2} \tag{5}$$

which simplifies to

$$(\frac{dW}{dz})^2 = \frac{iK^2\overline{R}}{\pi r(\overline{R}-r)} \tag{6}$$

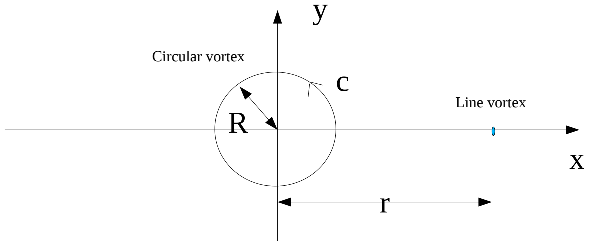

Figure 1: A cylinder in the presence of a line vortex. C represents the contour of integration for the complex potential of the combined flow.

The Blasius theorem can therefore be written

$$X - iY = -\frac{\rho K^2 \overline{R}}{2\pi r(\overline{R} - r)} \tag{7}$$

This then gives the forces

$$X = \frac{\rho K^2 \overline{R}}{2\pi r(r - \overline{R})}; Y = 0 \tag{8}$$

Since $\overline{R} < r$, the force is in the direction of positive x and so the cylinder is attracted towards the line vortex.

2.2 Circulation on the circle

Of course a hurricane has a very strong circulation associated with it. Let this circulation be given by K_h. In this case, the complex potential of the flow is given by

$$W = \frac{i}{2\pi}\{K(ln(z - r) - ln(\frac{R^2}{z} - r) + K_h lnz\} \tag{9}$$

Regrouping, differentiating and simplifying, we get additional residues which leads to the second term on the right hand side below

$$(\frac{dW}{dz})^2 = \frac{iK^2 \overline{R}}{\pi r(\overline{R} - r)} + \frac{iKK_h}{\pi r} \tag{10}$$

If we now rewrite the Blasius theorem for this case, we find

$$X - iY = -\frac{\rho K^2 \overline{R}}{2\pi r(\overline{R} - r)} - \frac{\rho KK_h}{2\pi r} \tag{11}$$

This then gives the forces

$$X = \frac{\rho K^2 \overline{R}}{2\pi r(r - \overline{R})} + \frac{\rho KK_h}{2\pi r}; Y = 0 \tag{12}$$

Now if we assume the circulation K to be associated with a sub-tropical high and therefore anticyclonic and K_h to be the circulation associated with a tropical storm and therefore cyclonic, K and K_h are of *opposite* sign. This leads to two regimes, one where there is an attractive force and one where the force repels with the transition point given by setting X=0 which leads to

$$|\frac{K}{K_h}| = \frac{r^2}{R^2} - 1 \tag{13}$$

Figure 2 shows the transition from attraction to repulsion in terms of storm radii for 3 values of the ratio of the two circulations. As can be seen, when a storm gets closer than a certain critical distance, the weak repulsion suddenly changes to strong attraction. Furthermore, the more concentrated the storm vortex, the more emphatic the switch between the two regimes. The diagram also suggests that strong hurricanes are perhaps more likely to be deflected as the force of repulsion is also larger. Weaker vortices have a smaller repulsion and the force changes to attraction further away from the line vortex so may be more likely to be attracted.

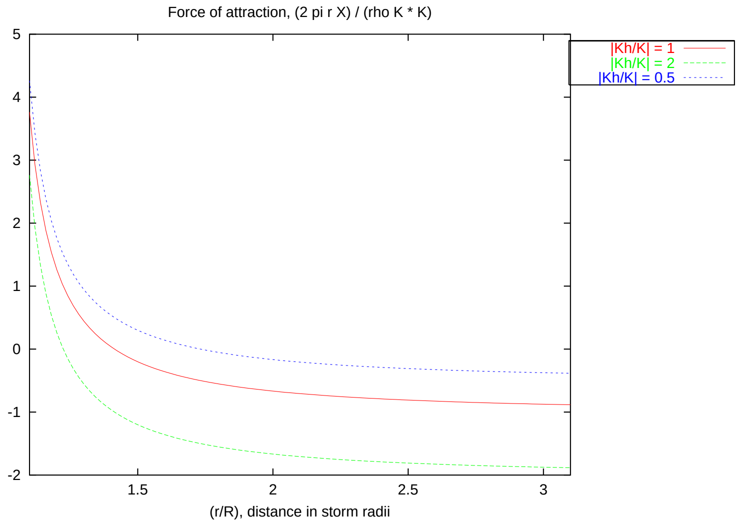

Figure 2: The normalised force $\frac{2\pi r X}{\rho K^2}$ as a separation distance between storm and sub-tropical high expressed as storm radii $\frac{r}{R}$

In part 2 of this study, we will analyse the movement of a cylindrical vortex in the field of a line vortex numerically.

References

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