Optimising the javelin throw in the presence of prevailing winds

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Abstract

Javelin flight is strictly governed by the laws of aerodynamics but there remain well-entrenched but misleading views about the factors involved. This paper gives a simplified but fundamentally accurate view of the physics and dynamics of javelin flight and describes a freely downloadable software implementation of these. The model has been calibrated against two sets of published competition data and predicts distances within about 1%. Finally, it goes on to show how throwing parameters can be optimised for given prevailing wind force and direction.

1 Overview

There have been numerous studies of the biomechanics of javelin throwing in the literature such as those by [1], [9] and [10] but there still appear to be many misunderstandings in practice about the actual aerodynamics of throwing in general and javelin throwing in particular in spite of particularly comprehensive treatments such as that by [5] and [2]. Other notable attempts to predict the distance a javelin flies from its release parameters include the work of [8] who use a neural net and they report accuracies of better than 5% and usually half this using this technique.

Of particular interest here is the effect of the prevailing wind on the javelin throw which is significant but has not been treated in such detail primarily because of a lack of controlled experiments in which it was measured. Given that a failure to understand the prevailing conditions and their impact on the throw can make all the difference between success and failure, this paper will attempt to address this deficiency. Many academic studies do not result in accessible technology so this paper will also address this. The physics will be described first followed by a mathematical model and finally by a description of a freely available calibrated modelling software package which embodies these principles, [4].

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The software models only post-1986 javelins. The essential difference was that the centre of gravity was moved forward in 1986 because of the prodigious distances being achieved by such throwers as Uwe Hohn, ($>$ 104m.). The only other alternative would have been to move the javelin outside the main arena which would have deprived onlookers of seeing one of the most spectacular of all field events. There have been other minor changes also but the effect of this re-balancing dominates and is essentially two-fold:-

- The average distance flown is reduced by around 10%
- Moving the centre of gravity forward means that it is now about 6cm in front of the *centre of pressure*. The centre of pressure is defined to be the point at which the aerodynamic forces of lift and drag on the javelin apply. This means that there is an upward lifting force 6cm aft of the downward force acting through the centre of gravity which is situated close to the front edge of the handle. This causes the javelin to rotate nose-downward.

  Note that this is something of a simplification because the centre of pressure moves during the flight although it does not move very much for a thin aerofoil such as the javelin, provided the angle of attack, (the angle between the javelin and the air-flow across it), remains small, [3]. This has been studied in detail in wind tunnel experiments using the javelins of the day by [2].

2 Some basic principles

The javelin is not so much a throw as a long pull. By far the most significant factor is the speed of the javelin as it leaves the hand, assuming it is aligned properly as the distance it travels is proportional to the square of the speed. In essence, the hand pulls on the javelin for a distance of some 2m in adult champion throwers accelerating the javelin from around 6 metres per second relative to the ground (allowing for the run-up) to around 30 metres per second relative to the ground.

3 The dynamics of the throw

Figure 1 shows the nomenclature used.

3.1 Simple mathematical treatment

This treatment gives a simple non-aerodynamic treatment. Any aerodynamic model of any complexity should degenerate to this as aerodynamic effects are removed so it is very useful as a sanity check in the more sophisticated model which follows later.

Let t_u be the time taken to reach the highest point of the flight from leaving the hand. Let t_d be the time taken to reach the ground from the highest point of the flight. Let s_u be the height reached above the point of release and let g

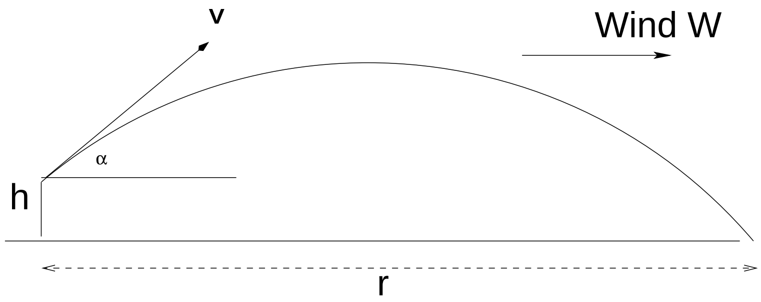

Figure 1: The nomenclature used in the accompanying text. A javelin is launched with velocity V at angle θ_0 from a height h m. above the ground with a wind of W m/s and flies a distance of r m.

be the acceleration due to gravity. Let h be the height at which the javelin is released. Let r be the distance covered by the javelin.

Then, resolving upward, the time taken to reach maximum height resisted by gravity is given by

$$0 = V sin\theta_0 - gt_u \tag{1}$$

Knowing V, θ_0 and g then gives t_u. The maximum height reached above the hand is given by:-

$$0 = V^2 sin^2\theta_0 - 2gs_u \tag{2}$$

The time taken to reach the ground from maximum height is given by

$$s_u + h = 0 + \frac{1}{2}gt_d^2 \tag{3}$$

Knowing h, s_u and g then gives t_d. The total time the javelin is in the air is therefore $t_u + t_d$. The horizontal distance covered by the javelin is therefore given by:-

$$r = V cos\theta_0(t_d + t_u) \tag{4}$$

This gives the classic parabolic shape of an ideal body in flight in a vacuum and is a useful approximation to the flight of a javelin. As will be seen in the experimental data presented later, the ideal case is only accurate to within about 5% for well-thrown javelins and to do it more accurately and account properly for wind, the aerodynamics must be re-introduced. Before introducing the aerodynamics, it is worthwhile commenting on three-dimensional effects. Although three dimensions is included in the model below, the direction perpendicular to the throw will be treated in a highly simplistic fashion as it is observed that in normal conditions good throws tend to stay in plane even though they may drift laterally. Predicting the range of poor throws seems of highly limited value and is not considered here.

3.2 Enhanced mathematical treatment including aerodynamics

Drag, lift and pitch First consider a thin aerofoil stationary in a horizontal wind as shown in Figure 2. For an aerofoil in an inviscid flow at angle β

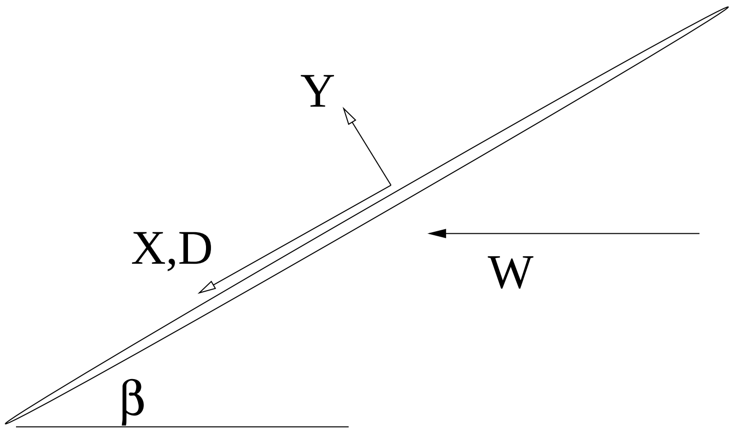

Figure 2: The aerodynamic forces acting on a thin aerofoil in a horizontal head wind of velocity W at an angle of β to that wind. The forces X and Y are the forces applied in an inviscid fluid generating net lift in the vertical direction but no drag and the force D is an approximation to allow for viscous effects, valid at small values of *beta*.

stationary to a horizontal wind of velocity U the forces X (along the aerofoil) and Y (perpendicular to the aerofoil) are given by, [3],

$$X = -2\pi\rho U sin\beta.K \tag{5}$$

$$Y = 2\pi\rho U cos\beta.K \tag{6}$$

where K is the circulation $= 2Usin\beta$, U is the velocity of the airflow and ρ is the density of air.

For inviscid flows, there is no drag but linearised theory for airflow around an object like a javelin based on a parabolic approximation of the nose shape ([14]) allows us to approximate the drag force when the javelin is parallel to the wind as that for a thin Joukowski airfoil.

$$D = 2\gamma\pi\rho U^2\epsilon^2 \tag{7}$$

where ϵ is the radius of the head of the javelin and γ is a constant. There is no lift in this case. Pitch (or attack) where the javelin makes a small angle with the wind can be simulated by increasing this value to represent more of the javelin's aspect being presented to the air-flow around it. A simple calculation reveals that the increased area presented assuming the javelin is cylindrical with radius ϵ (not a bad assumption given that its tapered at both ends and thicker in the middle) is given by

$$\pi\epsilon^2 + 4L\epsilon cos(\alpha) \tag{8}$$

where L is the length of the javelin and α is the angle of attack. The total longitudinal drag is therefore given approximately by

$$D \approx 2\gamma\rho U^2(\pi\epsilon^2 + 4L\epsilon cos(\alpha)) \tag{9}$$

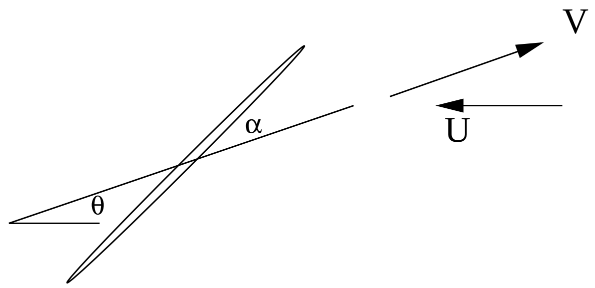

Figure 3: The angle θ is the direction of the centre of gravity of the javelin with respect to the ground and the angle α is the angle between the longitudinal axis of the javelin and the direction of movement of the centre of gravity. The javelin is launched with a velocity V into a head wind of strength U.

Javelin types Javelins come in three types, Headwind, Tailwind and neither (General), although not all manufacturers use precisely these terms. Javelin manufacturers are allowed very little flexibility in the position of the centre of gravity and it is normally checked at weigh-in. The only significant remaining parameter they have some control over is then the centre of pressure. The three kinds of javelin have the following properties:-

- Headwind. Pointed javelins which it is commonly believed fly a little further into a headwind but will not fly as well as a tailwind javelin into a tailwind.
- Tailwind. Blunter nosed javelins to attempt to break the boundary layer flow around the javelin a little earlier with the effect of bringing the centre of pressure a little closer to the centre of mass giving a smaller pitching moment to keep the nose up a little longer. The common belief is that these function better in a tail wind.
- General. These javelins are usually pointed and are not optimised in any particular way.

In the author's experience, folklore about the behaviour of these javelins abounds but the mathematical model which follows does not support the folklore which the author at first found puzzling. Certainly headwind and general javelins have less drag because the value of ϵ is smaller but the centre of pressure also moves so there is a counterbalancing effect. This was finally cleared up by [11] in a personal communication describing recent comments about Headwind and Tailwind javelins made by one of the pioneers of modern javelin design, Dick Held. In essence, Held made it clear that the javelins were originally distinguished only by a blunter nose. Apparently nobody would throw the blunt javelin however because it would obviously increase the drag, however *Held knew that the blunt javelin outperformed the sharp javelin in almost every environment*, (for very similar reasons to the higher performance of the rough-tailed javelin but not as emphatic). In order to get people to use it, he called it a Tailwind and people then began to use it in tail winds and so the belief grew.

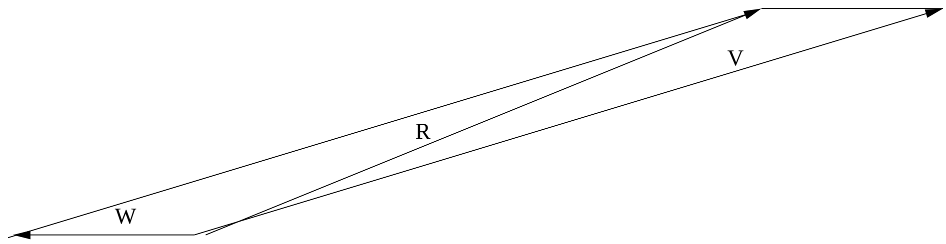

Figure 4: The parallelogram of velocities when a javelin is launched with velocity V at angle θ to the ground into a wind of W m/s. The resultant velocity is R

More on attack The above inviscid and viscous approximations now need to be combined and applied to a javelin configuration as shown in Figure 3 taking correct account of the relative motion between javelin and prevailing wind. This leads naturally to a discussion of attack.

Attack is rather more subtle than it appears. In aerodynamics, the angle of attack or *actual* angle of attack as it will be referred to here, is the angle α between the long axis of the javelin and the air-flow past the javelin. In previous studies such as that by [1], the angle of attack is measured as the angle between the long axis of the javelin and the direction of movement of the centre of gravity relative to the ground using a high speed camera. It is important to realise however that this is only the *apparent* angle of attack rather than the actual angle of attack. Previous authors have certainly been very well aware of this, [1], [2], but as will be seen here the difference is crucial when trying to account for prevailing wind conditions.

Consider 'throwing through the point' into the presence of a head wind as shown in Figure 4. The effect of the head wind is to reduce the velocity of the javelin but also *to change the direction of motion of its centre of gravity.* There is therefore a negative apparent attack angle relative to the ground even though the javelin was thrown through the point. Relative to the javelin, the air-flow is still parallel to the long axis of the javelin so the actual angle of attack is zero. Only when the prevailing wind is zero are the apparent angle of attack and the actual angle of attack the same. With non-zero winds they are not the same and it is perfectly possible for example, for the javelin to have a negative apparent angle of attack as filmed from the ground and a positive actual angle of attack generating lift (and also greater drag). It is of course the actual angle of attack which generates the aerodynamic forces of lift and additional drag.

Correcting for the relative motion In essence, correcting for the relative motion means applying a relative motion equal and opposite to V to Figure 3 so that it looks like Figure 2 so we can use the aerodynamic treatment shown earlier. The result is shown in Figure 5.

It can now be observed that Figure 5 can be separated into two figures which can both be mapped onto Figure 2. This is shown in Figure 6 where the right hand figure simply needs rotating through a right angle anticlockwise.

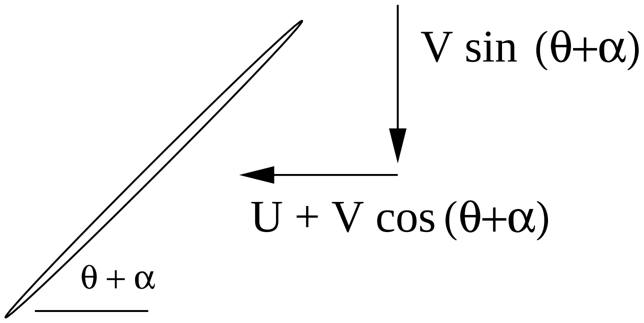

Figure 5: Transferring the frame of reference to the javelin by the application of a velocity equal and opposite to V.

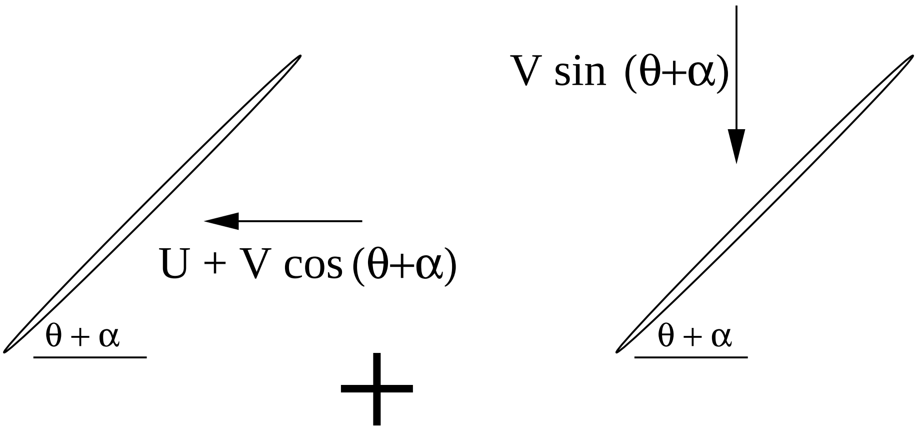

Figure 6: The decomposition of the relative motion corrected flow into two flows which can both be mapped onto Figure 2. The right hand figure simply needs rotating through 90 degrees anticlockwise. The lift and drag forces for each can then be combined to find the correct total lift and drag to apply in the equations of motion, corrected for the relative motion.

Full equations of motion This section simply turns the above physical factors into mathematical form so that they can be used to predict the flight of a javelin. This development assumes the javelin is delivered with no yaw or lateral actual angle of attack, (i.e. as seen from above) although allows for a cross-wind.

Let m be the mass of the javelin, α be the attack angle above the angle of delivery relative to the ground, (i.e. as seen by a fixed camera) and let the position vector of the javelin relative to the delivery line be (r,s,q) where r is in the direction of the javelin flight, s is upwards and q is across from right to left facing the javelin fan. A horizontal uniform wind of speed U will be assumed to be blowing from a constant heading ψ such that $\psi = 0$ degrees corresponds to a direct headwind and $\psi = 180$ degrees is a tailwind. Resolving in each of these three directions using Newton's law (mass x acceleration = Force) and including the inviscid force terms of equations (5) and (6) and the viscous drag term (9) corrected for relative motion to the ground as described in the Figure 6, leads to the following coupled non-linear differential equations:-

$$\begin{aligned} m\frac{d^2r}{dt^2} &= -2\gamma\rho(\frac{dr}{dt} + Ucos\psi)^2 cos(\theta+\alpha)\{\pi\epsilon^2 + 4L\epsilon cos\alpha\} \\ &\quad +4\pi\rho(\frac{dr}{dt})^2 sin^2(\theta+\alpha)cos(\theta+\alpha) \end{aligned} \tag{10}$$

$$\begin{aligned} m\frac{d^2s}{dt^2} &= -mg + 4\pi\rho(\frac{dr}{dt} + Ucos\psi)^2 sin(\theta+\alpha) \\ &\quad -2\gamma\rho(\frac{dr}{dt} + Ucos\psi)^2 sin(\theta+\alpha)\{\pi\epsilon^2 + 4L\epsilon cos\alpha\} \\ &\quad -2\gamma\rho(\frac{dr}{dt})^2 sin^3(\theta+\alpha)\{\pi\epsilon^2 + 4L\epsilon cos\alpha\} \end{aligned} \tag{11}$$

$$m\frac{d^2q}{dt^2} = -2\gamma\rho(\frac{dq}{dt} + Usin\psi)^2 cos(\theta+\alpha)\{\pi\epsilon^2 + 4L\epsilon cos\alpha\} \tag{12}$$

The two forces on the right hand side of equation (10) arise from the left and right parts of Figure 6 respectively. The second and third forces on the right hand side of equation (11) arise from the left hand part of Figure 6 and the fourth force from the right hand part of Figure 6.

Here γ is a fitting constant *which depends on the javelin type essentially through the point radius ϵ and partly through the shape.* These equations are integrated forward in time using a Kutta-Merson fourth-order procedure with control over global error, (see for example, [12]), to give the position vector (r,s,q) at all times after launch. If V is the initial launch speed, θ_0 the angle between the direction of movement of the centre of gravity at launch and ϕ_0 the azimuthal angle at launch, then the initial conditions at t = 0 are:-

$$r = 0 \tag{13}$$

$$\frac{dr}{dt} = Vcos\theta_0 cos\phi_0 - Ucos\psi \tag{14}$$

$$s = h \tag{15}$$

$$\frac{ds}{dt} = Vsin\theta_0 cos\alpha_0 \tag{16}$$

$$q = 0 \tag{17}$$

$$\frac{dq}{dt} = Vcos\theta_0 sin\phi_0 + Usin\psi \tag{18}$$

Pitching moment Finally the pitching moment is included by solving the angular pitching equation:-

$$mr\frac{d^2\theta}{dt^2} + 2mr\frac{d\theta}{dt} = -M \tag{19}$$

concurrently with the equations above, where M is the moment of the centre of gravity around the the centre of pressure. The movement of the centre of pressure is complex as it is related to the generally unknown behaviour of the boundary layer around the javelin during flight so this equation was parametrised for simplicity.

Rotation One factor which was omitted because it appears to be small is the axial rotation of the javelin. The axial rotation has been reported as high as 25 revolutions per second and for a right hander, coupled with the pitching moment, the javelin will precess slightly to the left because of the conservation of angular momentum. It is relatively easy to parametrise this and it will be included at a future stage.

Stiffness One other factor which was not incorporated is the effect of stiffness of the javelin. This has been considered in detail by [6], [7]. In the javelin throw, the throwing arm moves in an arc whilst holding onto the javelin. This bending can be substantial, particularly if the javelin is mis-hit. Although energy spent bending a javelin is ultimately dissipated as heat, it is an open question as to whether the vibrations affect drag and lift beneficially. The reader is referred to [6] for a detailed analysis and it will not be considered further here.

4 Effects of biomechanical factors

A considerable amount of work has been done on this using for example, the idea of kinetic chains by Bartlett, Morriss and others, [1], [9], [10]. In this simple analysis, a straight trade-off between the angle at which the javelin can be launched and the distance the thrower can hang on to it during the acceleration phase will be considered. The simplest representation of this is shown in Figure 7. In this diagram, using the cosine rule,

$$R^2 = 2L^2 - 2L^2 cos(180 - 2\theta) = 2L^2(1 + cos(2\theta)) \tag{20}$$

This can be simplified to give

$$R = 2Lcos(\theta) \tag{21}$$

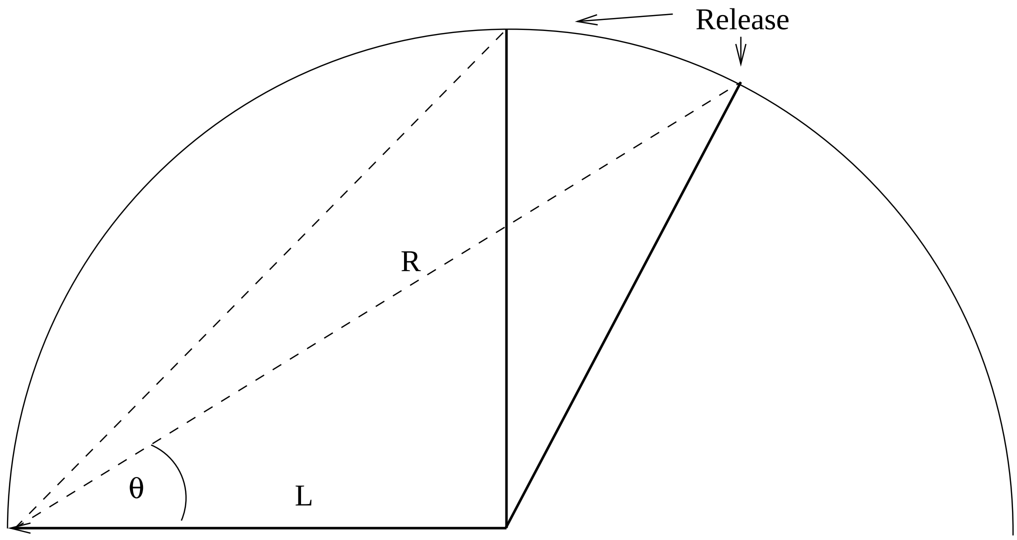

Figure 7: Illustrating the trade-off between increased angle of delivery θ and length of pull R which the javelin thrower can apply. Note that the movement forward of the shoulder pivoting on the base of the back and trunk can be modelled simply by increasing the 'length' of the arm, L.

So, as the delivery angle increases, the range along which the javelin can be pulled gradually diminishes. This is used in the software package itself to optimise the delivery parameters. It is noted in the paper by [2], page 385 that a linear decrease in release speed with increasing angle of release is observed for the range of release angles found in javelin throwing. Relative to the body, the relationship between the delivery velocity, arm acceleration and R using equation 21 is given by:-

$$V = \sqrt{(2aR)} = \sqrt{(4aLcos(\theta))} \tag{22}$$

Figure 8 shows the behaviour of the right hand side of equation 22 over a typical range of javelin delivery angles. As can be seen, it is very closely linear substantiating the results quoted in [2]. To gain an idea of the reduction in delivery speed with increasing angle, recall that the velocity V in equation 22 is relative to the body. For a delivery release of 30m/s, about 5m/s is due to the movement of the body so the final velocity achieved by the arm/shoulder action is about 25m/s of this. Substituting in equation 22 reveals a loss of around 0.124 m/s/degree for a throw launched at 30 degrees. This is in excellent agreement with the figure of 0.13 described by [13].

5 Modelling results

The above set of equations handle the qualitative behaviour of a javelin well and are included in the software package *Javelin Flight Analyser*, [4]. A screenshot of the front page is shown as Figure 9. The package is freely available as a self-installing executable for Windows 98/2000/XP from the quoted web location.

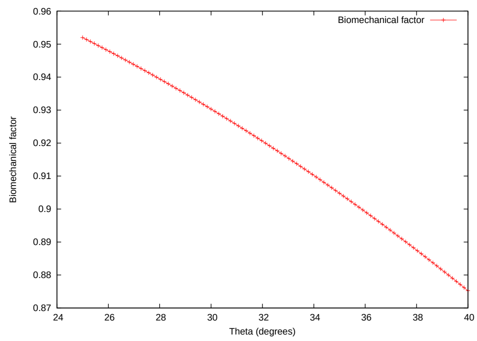

Figure 8: A plot of $\sqrt{(}4aLcos(\theta))$ against θ for a range of delivery angles typical for javelin throwing where 'a' is the maximum acceleration a thrower can apply and 'L' is the length of the thrower's arm.

5.1 Optimising flight in adverse conditions

Although solving the differential equations used in this model would once have been considered an expensive calculation, modern PCs are sufficiently fast as to be able to do this in a few milliseconds opening up the possibility of searching for optimal solutions in a given set of prevailing wind conditions or for fixed biomechanical factors for a particular athlete. A very simple example of this is shown in Figure 10 where the search was carried out simply by varying input parameters over typical ranges of values and recording the best. Note that the release velocity was fixed in this case. The software package can also allow this to be varied linearly with delivery angle to match the thrower's biomechanics in the manner described above.

5.2 Calibration

The model has been calibrated against the data recorded in the 1991 World student games as described by [1] and also some anonymous data from the 1993 BAF (British Athletic Federation) championships believed to have been acquired by Bartlett and Morriss but unattributed. These data collectively covered both male and female throwers with distances ranging between 57.22m and 87.42m.

As a control, Figure 11 shows the distances using the basic parabolic development of section 3.1 for the 1993 BAF championships. Using the sophisticated model described in section 3.2 and a gentle variable wind of -0.7 $\longrightarrow$ +0.9 m/s, (+ denoting a head wind), the fit of Figure 12 was achieved. Most throwers required a head wind to fit this well.

This was also done with the data quoted by [1] taken at the 1991 World Student Games. The equivalent wind-fitted data is shown in 13. In this case, the assumption of a small variable tail wind up to around 1.3 m/s was sufficient to produce this fit for the Men's event apart from one athlete where a small

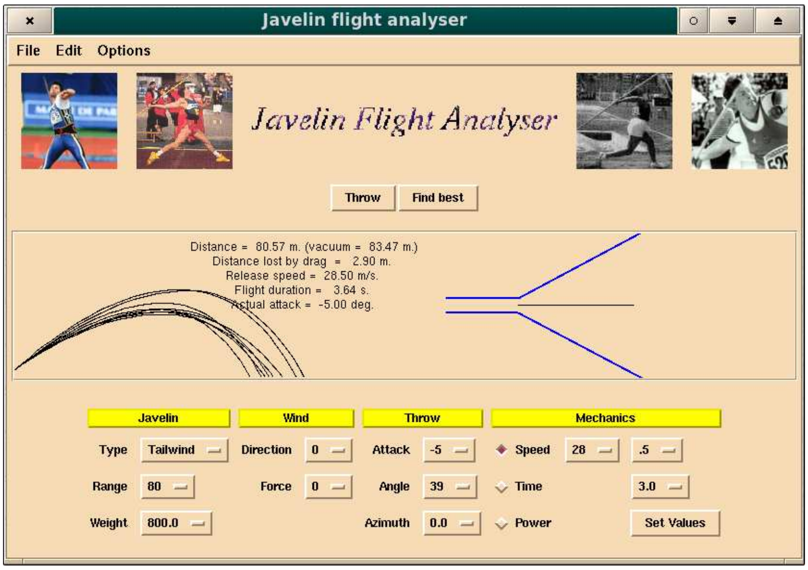

Figure 9: A screenshot of the front page of the Javelin Flight Analyser. The results of simulating the 1993 BAF championships are shown. These covered a range of delivery speeds between 25.4 and 28.5 metres/sec, attack angles of -5 to +7 degrees and delivery angles from 35 to 41 degrees. The very different nature of the trajectories and the non-parabolic effects of air resistance can be clearly seen.

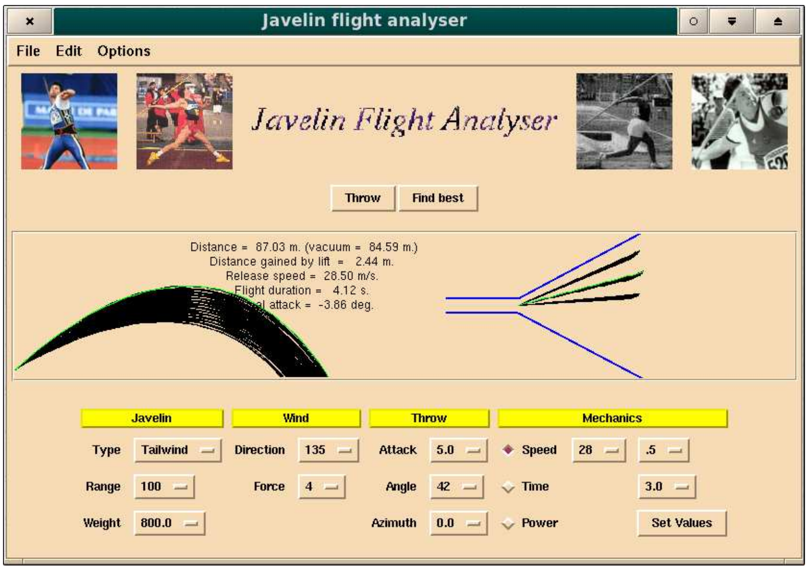

Figure 10: A screenshot of the front page of the Javelin Flight Analyser after performing an optimal search for a quartering wind.

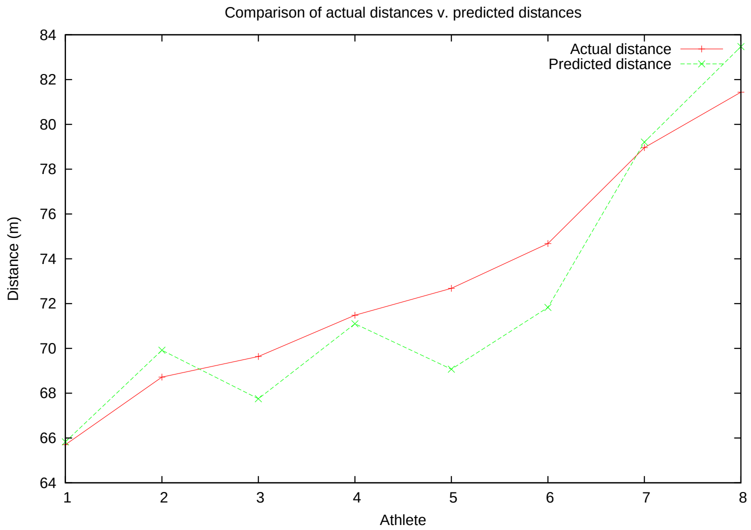

Figure 11: The result of predicting distances using the simple parabolic equations of section 3.1. The parabolic approximation is quite good with a maximum error of around 5% in this dataset.

headwind was necessary. It is intriguing to note that the wind assumptions are generally consistent for the event and reflect the conditions in many stadia competitions in which the author has taken part.

Note that this does not of course prove anything. However, no wind data is present for any of the quoted experiments to verify this more sophisticated calibration. It suffices to show here therefore that a gentle wind can account for all of the errors in the sophisticated model *without any other change to modelling parameters being necessary.* Moreover in each of the three cases, it led to a typical wind scenario which was consistent which gives some qualitative support for the approach used here. True confidence in this approach can only however spring from measurement and experiments of the nature of [1] will have to be repeated with careful measurement of the prevailing wind at the time of each performance as well as different left-foot plant positions to validate the model fully.

The model opens up the possibility of investigating scenarios in many different ways to find the optimum set of throwing parameters for a given prevailing wind and set of biomechanical restrictions. For example, in the case of Figure 10, if the thrower is capable of matching the specified parameters, (which can easily

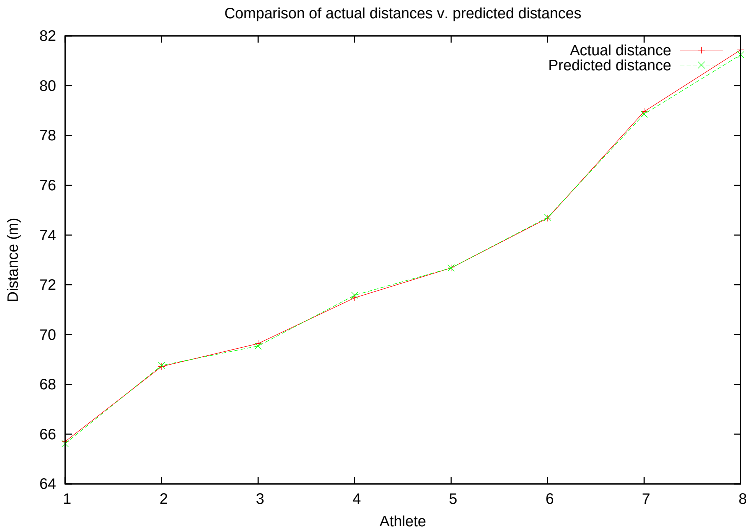

Figure 12: The result of predicting distances using the sophisticated coupled non-linear differential equations of section 3.2 including the effects of a gentle mostly head wind.

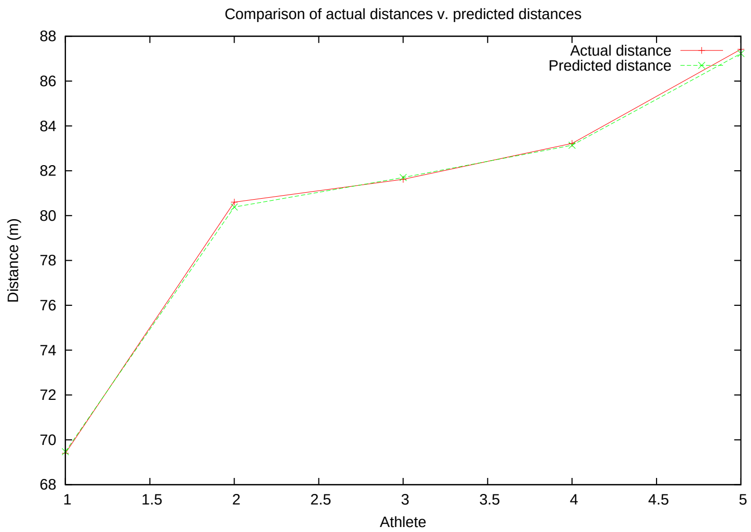

Figure 13: Wind fitted data for the sophisticated model of section 3.2 for the 1991 World Student's Games held in Sheffield, UK. This time a gentle tail wind was sufficient to provide a high degree of fit.

be restricted to match his or her biomechanical characteristics), the best throw is down the middle in this case with the attack angle shown. The nature of these optimisations will be explored in more detail when suitable calibration data for prevailing wind becomes available.

6 Conclusions

A sophisticated model of javelin flight including the effects of prevailing wind conditions has been presented along with a freely downloadable Windows program which implements it. The model has been calibrated against two sets of published data and produces good qualitative predictions for javelin flight in different wind conditions and is in good agreement with the small number of observations available in the literature. The model has proven to be capable of fitting the data to better than 1% under reasonable assumptions of wind but better data on prevailing wind conditions will be necessary to validate this level of accuracy.

The model also shows that it is not always obvious how to throw a javelin best in different conditions and there are some surprises for throwers. Finally it is clear that for wind-sensitive events like the javelin and discus, it is very important to understand the physics of the flight and the effects of the prevailing wind conditions on performance.

7 Acknowledgements

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