

Some comments on the teaching of mathematics

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1 A short commentary

I wrote this the day after the television presenter but most importantly of all, *celebrity*, Carol Vorderman was appointed by the Conservative Party to probe into maths education in the UK, (I use this phrase quite loosely). I have nothing whatsoever against Ms. Vorderman but to appoint somebody from the media with a degree in engineering, some competence in mental arithmetic on quiz shows but above all, as far as I am aware, absolutely no background whatsoever in the teaching of mathematics is little more than a sick joke.

This is important as there is a fair chance that the Conservative Party will form the next Government. I am a mathematician by training and have some experience teaching it. The major problem for the last 12 years is that the current Government has been ruthlessly spinning everything, including the impact of their educational policies, desperately trying to convince us that things have never been better. By the looks of the announcement above, we can expect more of the same.

Political interference and the humble logarithm Whatever the talking heads of Government might say, maths teaching in particular and STEM (Science, Technology, Engineering and Mathematics) teaching in general has suffered visibly. Both the current Government and the Opposition are jointly responsible for the fact that today, innumeracy is rife. Political interference, a total disregard for anybody who actually knows anything about it, and a half-baked political correctness which dictates that children shouldn't be given anything hard to do in case it damages them in some way, have jointly taken a terrible toll.

Let's take the humble, hard-working and totally neglected logarithm. Stay with me now. Many university students have never even heard of it as it has been removed from non-specialist parts of the curriculum presumably because it is considered difficult, and yet it is a beautiful example of a non-linear map. If you take two rulers calibrated normally, you can add and subtract by sliding one along the other. Just try it. With logarithmically calibrated rulers, you can actually multiply and divide. What could be more natural? These used to be called slide-rules before that Weapon of Math Destruction, the calculator was invented.

Are logarithms really so important ? Well, like many things in mathematics, it has a twin to which it is of course exceptionally closely related. For logarithms, that twin is called powers. If you don't know what these are, they look like this:-

$$3^2 = 3 \times 3 = 9 \quad (1)$$

Why on earth, would little numbers up on the top right be important ? Well, everybody in the developed world comes across these as they are the basis by which the mechanism of compound interest works, as in mortgages, loans and so on. If you don't understand them, you are effectively handing your trust over to the banks, and if any concept has taken a battering in the past couple of years, it is the idea that a bank can be trusted.

The calculator Let me expand a little more on the calculator. Calculators are almost single-handedly responsible for destroying two vitally important factors in teaching children useful mathematics without fear.

- The ability to do mental arithmetic. Most of the university students I come into contact with are truly hopeless at this. The importance of course of mental arithmetic is not the result, but the process of *visualisation* necessary to multiply say 13 by 17 successfully in your head. When you visualise in order to do mental arithmetic, you are beginning to leap from the specific to the general, the process known as abstraction and vital to the development of both logical thought and concentration.
- The ability to estimate. A little story here might help. I was sitting in a pizza emporium on a skiing trip with a whole bunch of family friends last year. I happened to be sitting on a table with a bunch of my neighbours' kids, all delightful company, and good students in good universities. We had been talking about how my generation has undermined their generation by emphasizing how examinations and standards were much lower today, implying that all their hard work was in some way devalued. They are right. They are just as bright as we were but our generation has fumbled the ball in passing along the skills given to us by my parent's generation. The bill came altogether so I idly asked them approximately how much should they pay each without looking at the calculator some of them had on their phones. Not only could they not easily answer but the mere act of asking them actually caused them distress. I could pass along many examples like this but I won't.

Being unable to estimate also makes the next generation much more susceptible to gross errors. For example, on numerous occasions, a shop-keeper has pressed the wrong button on a calculator and has come up with an answer out by an order of magnitude, (10 times too low or high). In general, because they are unable to estimate, they have no idea either that they have made an error or by how much.

The schools I have three children. They all went to different schools for a variety of reasons and in each of those schools, I had either to provide support, or in two cases, to take over completely. I had no particular wish that they should

become mathematicians, I just wanted to make sure they were not actively damaged by the schooling system. Most of my children's teachers were either not specialist mathematicians or did not have a particularly good grasp of English. In Britain, for the whole of my life, science, technology and mathematics have been downgraded as professions and the declining few there have been, generally can get good jobs outside of teaching now. The knock on effect of this is that mathematics teaching, particularly in the crucial primary years 5-11 is provided by people who are themselves often phobic about the subject.

My daughter is a wonderful example of what can happen. She came home one day about the age of 14 and announced to us that "she had been put in a less stressful maths group". On gently probing, it turns out that this group has significant chunks of the curriculum removed, presumably so it can't adversely affect the school's league table positions, a peculiarly British fixation. The argument the school gave when this was raised is that it would cause my daughter "less stress". I find this objectionable in the extreme. I intervened and helped her to work round the difficulties introduced by poor teaching and ultimately she got a very good A at A level even though she is pursuing a career in music. She is actually pretty good at maths when you take the trouble to work round the areas which are often so opaquely taught in schools. My other two children had similar experiences and finished up after intervention with equally good grades, although one an architect and the other, another musician.

I would like to say that this is entirely down to my teaching abilities but this would be completely untrue. It is simply what happens when children are given challenges in a secure way and carefully walked through things that initially seemed hard to them. They literally blossom before your eyes.

Mathematics is layered not modular Another modern misunderstanding is the growth of modular subjects. Mathematics has *never* been modular. It is layered. If a layer is neglected or incomplete, subsequent layers come crashing down sooner or later. It is now quite common to hear students saying that they did say, the calculus module last year and now they are doing other modules, they have completely forgotten it. It is impossible to progress unless the calculus is absorbed as a layer. To many people, the calculus is a complete black art assuming they have ever heard of it all. On the contrary, it is beautiful, simple to teach and of infinite application.

Another example is an understanding of probability. This has a vital part to play in assessing risk sensibly in an increasingly complex world. In spite of this, vast areas of society have absolutely no idea of its importance. Again it is not so hard to teach in the context of the risks and decisions naturally occurring in life, but like so many so-called "hard" subjects, we have effectively trashed its appeal and importance.

One of the implications of modularity is that mathematics is perceived as incoherent. In addition, because it is deemed hard, (by which we mean that conventionally taught students will get one grade less on average than if they had offered something like knitting, media studies, design or the like), they walk

away from it and do, no surprise here, knitting, media studies, design or the like. I do not wish to dismiss such subjects but they have no content which can help develop vital analytic skills. Similarly, I do not wish for a world populated by mathematicians - a disturbing thought in itself - but doing three A levels in media studies, design and origami has as much value in preparing students for life's complex challenges as a chocolate teapot would be for their warm-drink needs. A student with only these skills is simply cannon fodder for an increasingly rapacious and uncontrollably greedy financial sector. We and the politicians we elect owe them better than this.